Math 45 SSM 2/e

2.6 Direct Translation Involving Percent — Day 1

Day 1 { 1) Apply the (basic) percent equation

Day 2 { 2) Apply the percent increase equation
3) Apply the percent decrease equation

(Basic) Percent Equation

$$a = p \cdot b$$

a = amount
p = percent, as a decimal
b = base

"amount = percent of base"

The percent equation is equivalent to the percent proportion.

(Basic) Percent Proportion

$$\frac{a}{b} = \frac{p}{100}$$

a = amount
p = percent, NOT as a decimal
b = base

"amount is to base as percent is to 100"

- The percent equation is more closely related to the percent increase equation and percent decrease equation than the per cent proportion.
  Although either method is acceptable, I recommend the percent equation.

- To write a percent as a decimal, divide by 100
  eg $60\% = \frac{60}{100} = .6$
  So p and $\frac{p}{100}$ are the same, but different formats.

- To see how the equation and proportion are equivalent, multiply both sides of the proportion by b: $\frac{a}{b} \cdot b = \frac{p}{100} \cdot b \Rightarrow a = \frac{p}{100} \cdot b$
  (p as decimal!)

In basic percent problems, there are 3 different types of questions:

1) missing amount a [but p and b are known]
2) missing base b [but a and p are known]
3) missing percent p [but a and b are known].

To solve a basic percent problem:

Step 1: Identify p -- look for the % symbol or the words "what percent" tell you that p is unknown.

Step 2: Identify b -- look for the word "of" and b is the next number after "of", OR the words "of what" tell you that b is unknown.

Step 3: Identify a -- It's the last number you haven't used yet or the words "what is" tell you that a is unknown.

Step 4: Plug in the two known values in the percent equation, leaving a variable for the third.

Step 5: Isolate the variable using algebra.

Step 6: Write the answer. Check for units!

Examples

(1) What is 37% of 550?

$\begin{cases} p = .37 \\ b = 550 \\ a = \text{unknown} \end{cases}$ Identify values

$a = p \cdot b$ Write formula

$a = (.37)(550)$ Plug in

$\boxed{a = 203.5}$ Calculate

(2) 45% of what is 405?

$\begin{cases} p = .45 \\ b = \text{unknown} \\ a = 405 \end{cases}$ Identify values

$a = p \cdot b$ Write formula

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$405 = (.45)(b)$ Plug in

$\frac{405}{.45} = b$ Divide both sides by .45 to isolate b

$\boxed{b = 900}$

③ What percent of 400 is 60?

$\begin{cases} p = \text{unknown} \\ b = 400 \\ a = 60 \end{cases}$ Identify values

$a = p \cdot b$ Write formula

$60 = p \cdot 400$ Divide both sides by 400 to isolate p.

$\frac{60}{400} = p$ Divide and convert decimal back to %.

$p = .15 = \boxed{15\% = p}$

CAUTION The percent equation always has p written as a decimal. So you have to change it to a percent before writing your final answer.

Extra Practice (Solutions at end.)

④ 15 is 25% of what? $60 = b$ unknown

⑤ What is 30% of 70? 21, a

⑥ 135 is what percent of 27? $500\% = p$

⑦ What is 140% of 120? $168 = a$

⑧ 80 is what % of 60? $133\frac{1}{3}\% = p$

⑨ 45 is 180% of what? $25 = b$

Applications of the Basic Percent Equation

1) Commission
2) "% of" anything, e.g. Census, Exam Scores, Statistics

To work for commission is to sell something (cars, cell phones, appliances, real estate, etc.) and be paid a percentage of your sales.

→ Used as an incentive: sell more, get paid more!
→ Sometimes commission is your only pay: sell nothing, earn nothing
→ Sometimes you get a salary plus commission.

(10) Melanie earns 3% commission on every house she sells. If she earns $8571, what was the selling price of the house?

a = amount earned as commission = 8571
p = commission rate = 3% = .03
b = value of goods sold = unknown

$a = p \cdot b$ Write formula
$8571 = (.03)(p)$ Substitute values
$\frac{8571}{.03} = p$ Divide both sides by .03 to isolate p.

$\boxed{p = \$285,700}$ Write answer with units.

(11) The Census reports that 29% of the 106 million males 18 or over have never married. How many males 18 or over have never married?

$\begin{cases} a = \text{unknown} \\ p = .29 \\ b = 106 \text{ million} \end{cases}$

$a = p \cdot b$
$a = (.29)(106)$
$= (30.74)(1,000,000)$
$\boxed{\begin{array}{l} a = 30.74 \text{ million} \\ a = 30,740,000 \end{array}}$

(12) In an election for school president, the loser received 60% of the winner's votes. If 848 votes were cast, how many did each receive? (Assume only two candidates.)

x = # votes for winner
$.6x$ = # votes for loser

Note: x is just $1x$

$x + .6x = 848$
$1.6x = 848$
$x = \frac{848}{1.6}$
$\boxed{x = 530 \text{ votes for winner}}$

$.6x = (.6)(530)$
$= \boxed{318 \text{ votes for loser}}$

(13) Mario is paid on commission for ad sales. He earns 8% on $450 full-page ads and 5% on $300 half-page ads. If he sells twice as many half-page ads as full-page ads, and his commission is $5610, how many of each type did he sell?

x = # full-page ads sold.
$2x$ = # half-page ads sold. (Twice as many)

money for full-page ads $= 450x$
money for half-page ads $= 300 \cdot 2x = 600x$

commission for full-page $= 8\%$ of $450x$
$= (.08)(450x)$
$= 36x$

commission for half-page $= 5\%$ of $600x$
$= (.05)(600x)$
$= 30x$

Add commissions to get total commission:

$$36x + 30x = 5610$$

$$66x = 5610 \qquad \text{combine}$$

$$x = \frac{5610}{66} \qquad \text{isolate } x$$

$$\boxed{x = 85 \text{ full-page-ads}}$$

$2x = 2 \cdot 85$

$$\boxed{2x = 170 \text{ half-page ads}}$$

Solutions to Extra Practice

(4) 15 is 25% of what?

$\begin{cases} p = .25 \\ b = \text{unknown} \\ a = 15 \end{cases}$

$a = p \cdot b$

$15 = .25\, b$

$\frac{15}{.25} = b$

$$\boxed{b = 60}$$

⑤ What is 30% of 70?

$\begin{cases} p = .3 \\ b = 70 \\ a = \text{unknown} \end{cases}$

$a = p \cdot b$

$a = (.3)(70)$

$\boxed{a = 21}$

⑥ 135 is what % of 27?

$\begin{cases} p = \text{unknown} \\ b = 27 \\ a = 135 \end{cases}$

$a = p \cdot b$

$135 = p \cdot 27$

$\frac{135}{27} = p$

$p = 5 = \boxed{500\%}$

Note: If a is larger than b, percent will be larger than 100%

⑦ What is 140% of 120?

$\begin{cases} p = 1.4 \\ b = 120 \\ a = \text{unknown} \end{cases}$

$a = p \cdot b$

$a = (1.4)(120)$

$\boxed{a = 168}$

⑧ 80 is what % of 60?

$\begin{cases} p = \text{unknown} \\ b = 60 \\ a = 80 \end{cases}$

$a = p \cdot b$

$80 = p \cdot 60$

$\frac{80}{60} = p$

$p = 1.\overline{3} = \boxed{133.\overline{3}\% = 133\frac{1}{3}\%}$

* CAUTION *

DO NOT ROUND UNLESS INSTRUCTIONS SAY TO ROUND. USE FRACTIONS

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(9) 45 is 180% of what?

$$\begin{cases} p = 1.8 \\ b = \text{unknown} \\ a = 45 \end{cases}$$

$a = p \cdot b$

$45 = 1.8 \cdot b$

$\frac{45}{1.8} = b$

$\boxed{b = 25}$